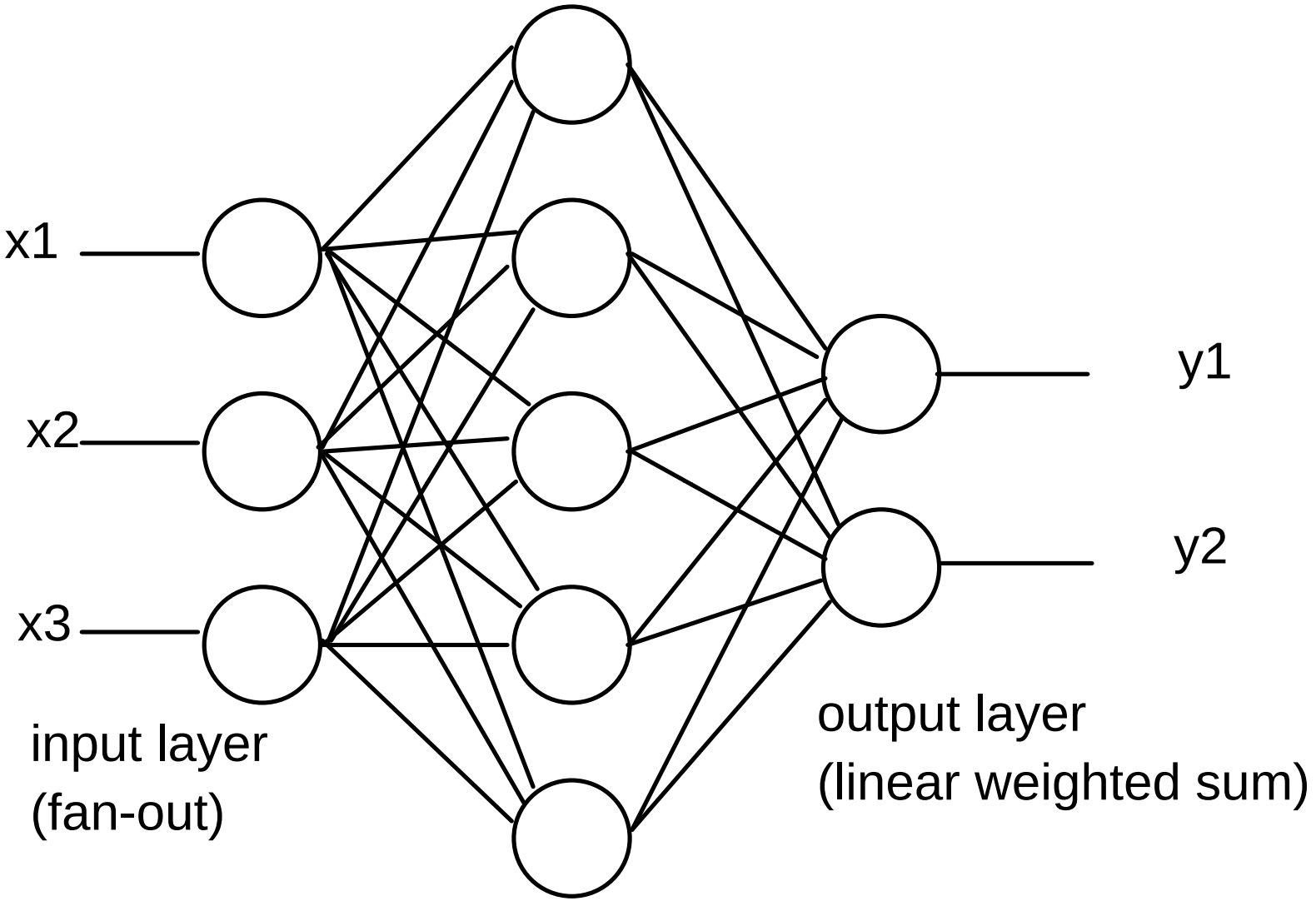
Radial Basis Function (RBF) Networks

RBF network

- This is becoming an increasingly popular neural network with diverse applications and is probably the main rival to the multi-layered perceptron
- Much of the inspiration for RBF networks has come from traditional statistical pattern classification techniques

RBF network

- The basic architecture for a RBF is a 3-layer network, as shown in Fig.
- The input layer is simply a fan-out layer and does no processing.
- The second or hidden layer performs a non-linear mapping from the input space into a (usually) higher dimensional space in which the patterns become linearly separable.


x1
x2
x3
y1
y2
input layer
(fan-out)
output layer
(linear weighted sum)
hidden layer
(weights correspond to cluster centre,
output function usually Gaussian)

Output layer

- The final layer performs a simple weighted sum with a linear output.
- If the RBF network is used for function approximation (matching a real number) then this output is fine.
- However, if pattern classification is required, then a hard-limiter or sigmoid function could be placed on the output neurons to give 0/1 output values.

Clustering

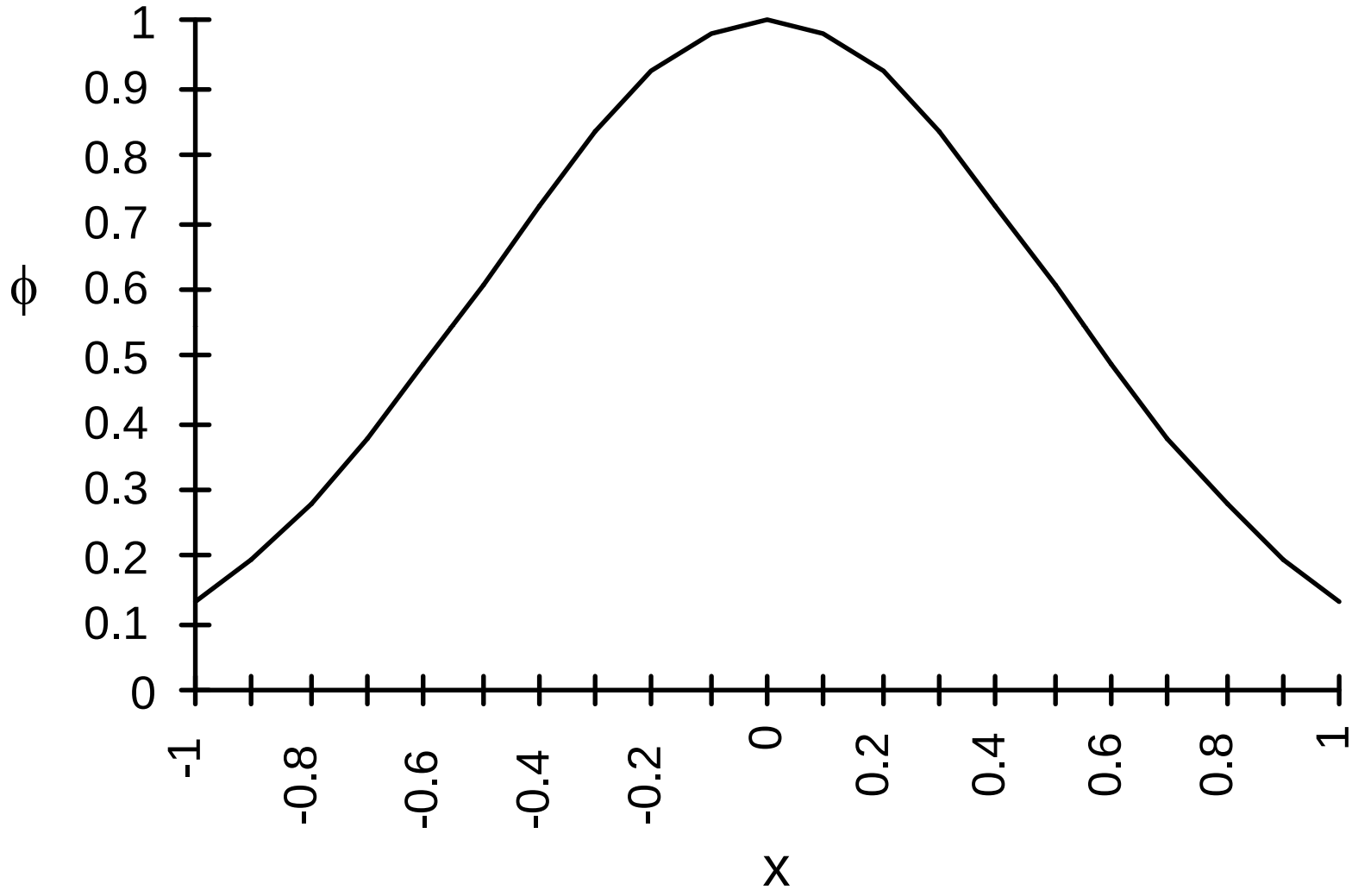
- The unique feature of the RBF network is the process performed in the hidden layer.
- The idea is that the patterns in the input space form clusters.
- If the centres of these clusters are known, then the distance from the cluster centre can be measured.
- Furthermore, this distance measure is made non-linear, so that if a pattern is in an area that is close to a cluster centre it gives a value close to 1.

Clustering

- Beyond this area, the value drops dramatically.
- The notion is that this area is radially symmetrical around the cluster centre, so that the non-linear function becomes known as the radial-basis function.

Gaussian function

- The most commonly used radial-basis function is a Gaussian function
- In a RBF network, *r* is the distance from the cluster centre.
- The equation represents a Gaussian bell-shaped curve, as shown in Fig.


1
0.9
0.8
0.7
ϕ 0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x

Distance measure

- The distance measured from the cluster centre is usually the Euclidean distance.
- For each neuron in the hidden layer, the weights represent the co-ordinates of the centre of the cluster.
- Therefore, when that neuron receives an input pattern, *X*, the distance is found using the following equation:

Distance measure

$$r_j = \sqrt{\sum_{i=1}^{n} (x_i - w_{ij})^2}$$

Width of hidden unit basis function

$$(hidden\_unit)\varphi_j = \exp\left(-\frac{\sum_{i=1}^{n}(x_i - w_{ij})^2}{2\sigma^2}\right)$$

The variable sigma, σ, defines the width or radius of the bell-shape and is something that has to be determined empirically. When the distance from the centre of the Gaussian reaches σ, the output drops from 1 to 0.6.

Example

- An often quoted example which shows how the RBF network can handle a non-linearly separable function is the exclusive-or problem.
- One solution has 2 inputs, 2 hidden units and 1 output.
- The centres for the two hidden units are set at c1 = 0,0 and c2 = 1,1, and the value of radius σ is chosen such that $2\sigma^2 = 1$.

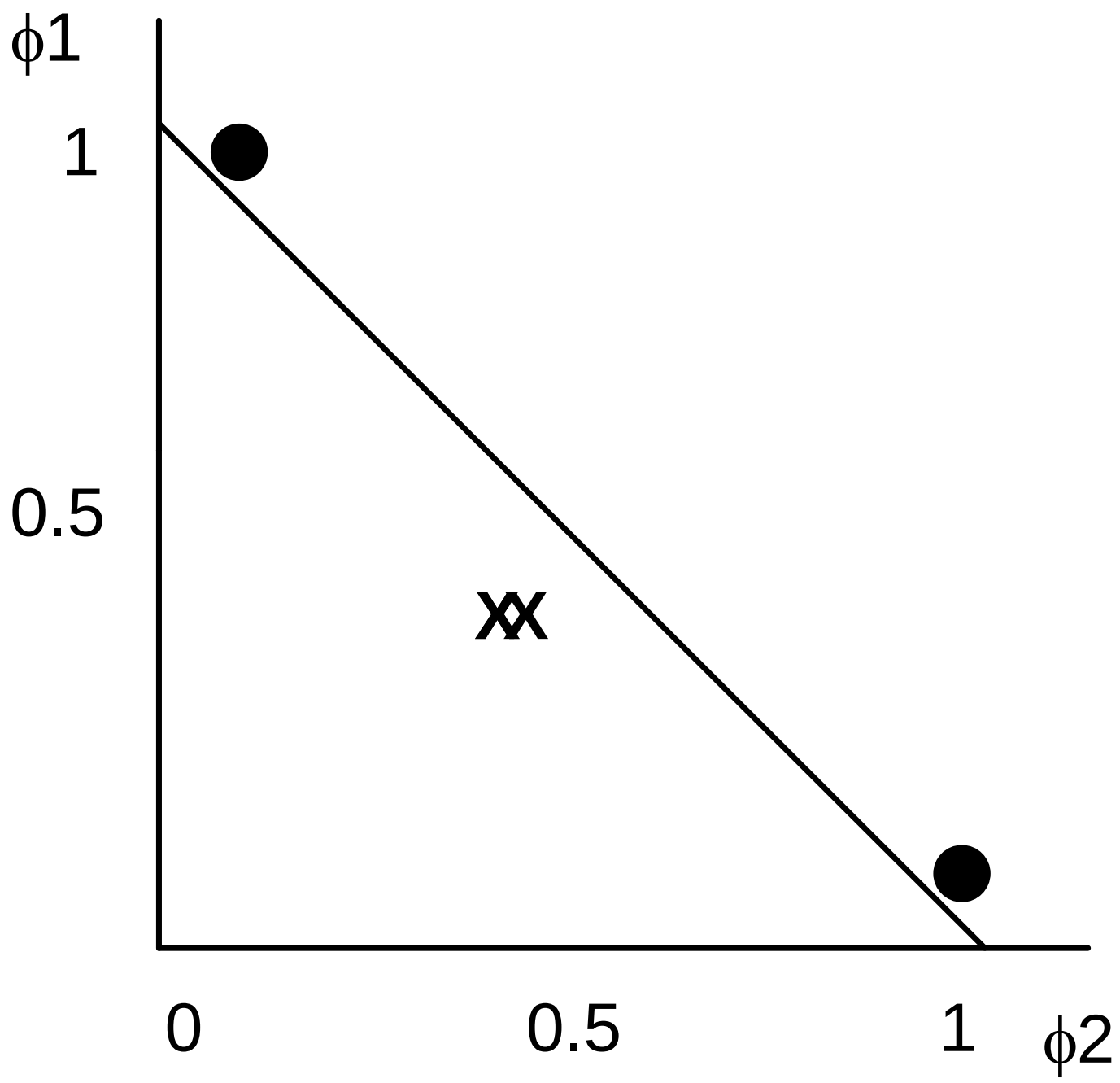
Example

- The inputs are x, the distances from the centres squared are r, and the outputs from the hidden units are φ.
- When all four examples of input patterns are shown to the network, the outputs of the two hidden units are shown in the following table.

x_1	x_2	r_1	r_2	φ_1	φ_2
0	0	0	2	1	0.1
0	1	1	1	0.4	0.4
1	0	1	1	0.4	0.4
1	1	2	0	0.1	1

Example

- Next Fig. shows the position of the four input patterns using the output of the two hidden units as the axes on the graph - it can be seen that the patterns are now linearly separable.
- This is an ideal solution - the centres were chosen carefully to show this result.
- Methods generally adopted for learning in an RBF network would find it impossible to arrive at those centre values - later learning methods that are usually adopted will be described.


ϕ1
1
0.5
0
0.5
1
ϕ2

Training hidden layer

- The hidden layer in a RBF network has units which have weights that correspond to the vector representation of the centre of a cluster.
- These weights are found either using a traditional clustering algorithm such as the *k*-means algorithm, or adaptively using essentially the Kohonen algorithm.

Training hidden layer

- In either case, the training is unsupervised but the number of clusters that you expect, *k*, is set in advance. The algorithms then find the best fit to these clusters.
- The *k* -means algorithm will be briefly outlined.
- Initially *k* points in the pattern space are randomly set.

Training hidden layer

- Then for each item of data in the training set, the distances are found from all of the *k* centres.
- The closest centre is chosen for each item of data - this is the initial classification, so all items of data will be assigned a class from 1 to *k*.
- Then, for all data which has been found to be class 1, the average or mean values are found for each of co-ordinates.

Training hidden layer

- These become the new values for the centre corresponding to class 1.
- Repeated for all data found to be in class 2, then class 3 and so on until class *k* is dealt with - we now have *k* new centres.
- Process of measuring the distance between the centres and each item of data and re-classifying the data is repeated until there is no further change – i.e. the sum of the distances monitored and training halts when the total distance no longer falls.

Adaptive k-means

- The alternative is to use an adaptive *k* -means algorithm which similar to Kohonen learning.
- Input patterns are presented to all of the cluster centres one at a time, and the cluster centres adjusted after each one. The cluster centre that is nearest to the input data wins, and is shifted slightly towards the new data.
- This has the advantage that you don’t have to store all of the training data so can be done on-line.

Finding radius of Gaussians

- Having found the cluster centres using one or other of these methods, the next step is determining the radius of the Gaussian curves.
- This is usually done using the *P*-nearest neighbour algorithm.
- A number *P* is chosen, and for each centre, the *P* nearest centres are found.

Finding radius of Gaussians

- The root-mean squared distance between the current cluster centre and its P nearest neighbours is calculated, and this is the value chosen for σ.
- So, if the current cluster centre is c_j, the value is:

$$\sigma_j = \sqrt{\frac{1}{P}\sum_{i=1}^{P}(c_k - c_i)^2}$$

Finding radius of Gaussians

- A typical value for P is 2, in which case σ is set to be the average distance from the two nearest neighbouring cluster centres.

XOR example

- Using this method XOR function can be implemented using a minimum of 4 hidden units.
- If more than four units are used, the additional units duplicate the centres and therefore do not contribute any further discrimination to the network.
- So, assuming four neurons in the hidden layer, each unit is centred on one of the four input patterns, namely 00, 01, 10 and 11.

- The *P*-nearest neighbour algorithm with *P* set to 2 is used to find the size if the radii.
- In each of the neurons, the distances to the other three neurons is 1, 1 and 1.414, so the two nearest cluster centres are at a distance of 1.
- Using the mean squared distance as the radii gives each neuron a radius of 1.
- Using these values for the centres and radius, if each of the four input patterns is presented to the network, the output of the hidden layer would be:

input	neuron 1	neuron 2	neuron 3	neuron 4
00	0.6	0.4	1.0	0.6
01	0.4	0.6	0.6	1.0
10	1.0	0.6	0.6	0.4
11	0.6	1.0	0.4	0.6

Training output layer

- Having trained the hidden layer with some unsupervised learning, the final step is to train the output layer using a standard gradient descent technique such as the Least Mean Squares algorithm.
- In the example of the exclusive-or function given above a suitable set of weights would be +1, –1, –1 and +1. With these weights the value of *net* and the output is:

input	neuron 1	neuron 2	neuron 3	neuron 4	*net*	output
00	0.6	0.4	1.0	0.6	–0.2	0
01	0.4	0.6	0.6	1.0	0.2	1
10	1.0	0.6	0.6	0.4	0.2	1
11	0.6	1.0	0.4	0.6	–0.2	0

Advantages/Disadvantages

- RBF trains faster than a MLP
- Another advantage that is claimed is that the hidden layer is easier to interpret than the hidden layer in an MLP.
- Although the RBF is quick to train, when training is finished and it is being used it is slower than a MLP, so where speed is a factor a MLP may be more appropriate.

Summary

- Statistical feed-forward networks such as the RBF network have become very popular, and are serious rivals to the MLP.
- Essentially well tried statistical techniques being presented as neural networks.
- Learning mechanisms in statistical neural networks are not biologically plausible – so have not been taken up by those researchers who insist on biological analogies.